

Scale dependence of f_{NL} in N-flation

Qing-Guo Huang *

Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics,
Chinese Academy of Sciences, Beijing 100190, China

ABSTRACT: Adopting the horizon-crossing approximation, we derive the spectral index of f_{NL} in general N-flation model. Axion N-flation model is taken as a typical model for generating a large f_{NL} which characterizes the size of local form bispectrum. We find that its tilt $n_{f_{NL}}$ is negligibly small when all inflatons have the same potential, but a negative detectable $n_{f_{NL}}$ can be achieved in the axion N-flation with different decay constants for different inflatons. The measurement of $n_{f_{NL}}$ can be used to support or falsify the axion N-flation in the near future.

KEYWORDS: N-flation, non-Gaussianity.

*huangqg@itp.ac.cn

Contents

| | |
|---|-----------|
| 1. Introduction | 1 |
| 2. Scale dependence of f_{NL} in general N-flation | 2 |
| 3. Axion N-flation | 5 |
| 3.1 Axion N-flation with the same potential for different inflatons | 6 |
| 3.1.1 $N_* \simeq N_{tot}$ | 7 |
| 3.1.2 A few inflatons stays around the hilltop | 9 |
| 3.2 Axion N-flation with different potential for different inflatons | 10 |
| 3.2.1 One inflaton with different decay constant stays around the hilltop | 10 |
| 3.2.2 A general axion N-flation with two kinds of inflatons | 11 |
| 4. Discussions | 14 |
| A. The trispectrum in N-flation | 15 |

1. Introduction

In the single-component slow-roll inflation the different Fourier modes of the curvature perturbation are roughly uncorrelated and their distribution is almost Gaussian [1]. Using δN formalism [2], the curvature perturbation can be expanded to the non-linear orders

$$\zeta = \delta N = \sum_i N_{,i} \delta\phi_i + \frac{1}{2} \sum_{i,j} N_{,ij} \delta\phi_i \delta\phi_j + \frac{1}{6} \sum_{i,j,k} N_{,ijk} \delta\phi_i \delta\phi_j \delta\phi_k + \dots, \quad (1.1)$$

where the subscript $,i$ denotes the derivative with respect to ϕ_i . The above formula indicates that the curvature perturbation is determined by the evolution of the homogeneous universe corresponding both to the classical inflation trajectory and to nearby trajectories. In principle, an important difference for the multi-field model from the single-component case is that the classical trajectory is not uniquely specified by the potential, but rather has to be given as a separate piece of information. It provides an opportunity to generate a large non-Gaussianity in multi-field slow-roll inflation even at the inflationary epoch [3, 4, 5], not at the end of inflation [6, 7, 8]. See some other relevant references in [9, 10, 11, 12, 13, 14, 15, 17, 16, 18].

On the other hand, not only the size of bispectrum f_{NL} is measurable, but also its scale dependence characterized by $n_{f_{NL}}$ is possibly measured by the accurate cosmological observations, such as Planck and CMBPol. In [19] the authors concluded that Planck and

CMBPol are able to provide a $1\text{-}\sigma$ uncertainty on the spectral index of f_{NL} for local form bispectrum as follows

$$\Delta n_{f_{NL}} \simeq 0.1 \frac{50}{f_{NL}} \frac{1}{\sqrt{f_{sky}}} \quad \text{for Planck,} \quad (1.2)$$

and

$$\Delta n_{f_{NL}} \simeq 0.05 \frac{50}{f_{NL}} \frac{1}{\sqrt{f_{sky}}} \quad \text{for CMBPol.} \quad (1.3)$$

Recently the scale dependence of f_{NL} for inflation and curvaton model is discussed in [20, 21, 22, 23, 24] and a large parameter space for generating a detectable $n_{f_{NL}}$ in curvaton model is illustrated in [22, 23]. The value of $n_{f_{NL}}$ will be an important discriminator to distinguish different models.

In this paper we focus on a class of multi-field inflation, so-called N-flation, in which there are no cross couplings among the inflaton fields or the cross couplings are negligibly small. For an example, we consider the axion N-flation with a detectably large f_{NL} . But we find that $n_{f_{NL}}$ is negligibly small when all inflatons have the same potential. However, a negative $n_{f_{NL}}$ with detectably large absolute value can be obtained in the model with different decay constants for different inflatons.

Our paper is organized as follows. In Sec. 2 we derive the spectral index of f_{NL} for general N-flation. We explore the observables, including $n_{f_{NL}}$, in the axion N-flation in Sec. 3. Some discussions are contained in Sec. 4.

2. Scale dependence of f_{NL} in general N-flation

In this section we will derive the spectral index of f_{NL} in general N-flation with N_f uncoupled inflaton fields. The potential of inflatons is given by

$$V = \sum_{i=1}^{N_f} V_i(\phi_i). \quad (2.1)$$

The dynamics of inflation is governed by

$$H^2 = \frac{1}{3} \sum_{i=1}^{N_f} \left(\frac{1}{2} \dot{\phi}_i^2 + V_i(\phi_i) \right), \quad (2.2)$$

$$\ddot{\phi}_i + 3H\dot{\phi}_i = -V'_i, \quad (2.3)$$

where $V'_i = \partial V_i / \partial \phi_i$. In this paper we work on the unit of $M_p = 1$. In the slow-roll limit, the above equations are simplified to be

$$H^2 \simeq \frac{V}{3}, \quad (2.4)$$

$$3H\dot{\phi}_i \simeq -V'_i. \quad (2.5)$$

Similarly, we introduce some ‘slow-roll’ parameters, such as

$$\epsilon_i \equiv \frac{1}{2} \left(\frac{V'_i}{V_i} \right)^2. \quad (2.6)$$

From Eq.(2.4), we find

$$\dot{H} = -\frac{1}{6} \sum_i V_i'^2 / V, \quad (2.7)$$

and then

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2} \frac{\sum_i V_i'^2}{V^2}, \quad (2.8)$$

or equivalently,

$$\epsilon_H = \sum_i \left(\frac{V_i}{V} \right)^2 \epsilon_i. \quad (2.9)$$

Inflation happens when $\epsilon_H < 1$. The multi-field slow-roll inflation can be achieved even when some inflaton fields have steep potentials because the friction term for each inflaton is contributed from all of fields.

Following [25], the number of e-folds before the end of inflation is given by

$$N = \sum_{i=1}^{N_f} \int_{\phi_i^{end}}^{\phi_i} \frac{V_i}{V_i'} d\phi_i. \quad (2.10)$$

Therefore we obtain

$$N_{,i} = \frac{V_i}{V_i'}, \quad (2.11)$$

and

$$N_{,ij} = \left(1 - \frac{V_i V_i''}{V_i'^2} \right) \delta_{ij}. \quad (2.12)$$

Using the δN formalism [2], the power spectrum of scalar perturbation is given by

$$P_\zeta = \left(\frac{H}{2\pi} \right)^2 \sum_i N_{,i}^2. \quad (2.13)$$

A slow-roll inflation model predicts a near scale-invariant power spectrum of scalar perturbation. The deviation from exact scale invariance is described by the spectral index n_s which is defined by

$$n_s \equiv 1 + \frac{d \ln P_\zeta}{d \ln k} = 1 - 2\epsilon_H - 2 \frac{\sum_{i,j} \epsilon_{ij} N_{,i} N_{,j}}{\sum_k N_{,k}^2}, \quad (2.14)$$

where

$$\epsilon_{ij} \equiv \frac{V_i'}{V} \frac{V_j'}{V} - \frac{V_i''}{V} \delta_{ij}. \quad (2.15)$$

See [25, 26] in detail. In this paper, we assume, without loss of generality, $\dot{\phi}_i < 0$, so that $V'_i > 0$ for $\forall i$. On the other hand, the gravitational wave perturbation is also generated during inflation and the amplitude of its power spectrum is determined by the inflation scale

$$P_T = \frac{H^2}{\pi^2/2}, \quad (2.16)$$

and then the tensor-scalar ratio r_T becomes

$$r_T \equiv P_T/P_\zeta = 16 \left/ \sum_i \frac{1}{\epsilon_i} \right. . \quad (2.17)$$

The amplitude of gravitational perturbation is also near scale-invariant and its tilt is defined by

$$n_T \equiv \frac{d \ln P_T}{d \ln k} = -2\epsilon_H. \quad (2.18)$$

In single-field inflation, $r_T = 16\epsilon_H$ which leads to a consistency relation $n_T = -r_T/8$. For N -field inflation, it is modified to be

$$n_T \leq -r_T/8. \quad (2.19)$$

Here we used the inequality

$$\epsilon_H \geq 1 \left/ \sum_i \frac{1}{\epsilon_i} \right. , \quad (2.20)$$

where the equality is satisfied when $\epsilon_i = \epsilon_H V/V_i$ for $\forall i$.

From Eq.(1.1), the non-Gaussianity parameter f_{NL} can be written by

$$f_{NL} = \frac{5}{6} \sum_{i,j} \frac{N_{,i} N_{,j} N_{,ij}}{(\sum_k N_{,k}^2)^2}. \quad (2.21)$$

Since $N_{,ij} \propto \delta_{ij}$ for N -flation, the above formula is simplified to be

$$f_{NL} = \sum_i f_{NL}^i, \quad (2.22)$$

where

$$f_{NL}^i = \frac{5}{6} \frac{N_{,i}^2 N_{,ii}}{(\sum_k N_{,k}^2)^2} = \frac{5}{6} \frac{r_T^2}{128 \epsilon_i} \left(1 - \frac{V_i V_i''}{V_i'^2} \right). \quad (2.23)$$

Following [21], the spectral index of f_{NL} for N -flation is

$$n_{f_{NL}} = \frac{1}{f_{NL}} \sum_i f_{NL}^i (2n_{multi,i} + n_{f,i}), \quad (2.24)$$

where

$$n_{multi,i} = 2 \sum_{k,l} \epsilon_{kl} \left(\frac{N_{,k} N_{,l}}{\sum_j N_{,j}^2} - \delta_{il} \frac{N_{,k}}{N_{,i}} \right), \quad (2.25)$$

$$n_{f,i} = - \sum_k \frac{N_{,k} F_{kii}^{(2)}}{N_{,ii}}, \quad (2.26)$$

and

$$F_{kii}^{(2)} = -2 \left(\frac{V'_i}{V} \right)^2 \frac{V'_k}{V} + \frac{V''_i}{V} \frac{V'_k}{V} + 2 \frac{V'_i}{V} \frac{V''_i}{V} \delta_{ki} - \frac{V'''_i}{V} \delta_{ki}. \quad (2.27)$$

Considering $N_{,i} = V_i/V'_i$, we obtain

$$n_{multi,i} = (1 - n_s - 2\epsilon_H) - 4 \frac{V_i}{V} \epsilon_i \left(1 - \frac{V_i V''_i}{V'^2_i} \right), \quad (2.28)$$

$$n_{f,i} = 4 \left(\frac{V_i}{V} \right)^2 \epsilon_i - \frac{V_i}{V} \left(\frac{V''_i}{V_i} - \frac{V'''_i}{V'_i} \right) \bigg/ \left(1 - \frac{V_i V''_i}{V'^2_i} \right). \quad (2.29)$$

Using Eqs.(2.22), (2.23), (2.24), (2.28) and (2.29), we can easily calculate the spectral index of f_{NL} in general N-flation. The first term in (2.28) contribute $2(1 - n_s - 2\epsilon_H)$ to $n_{f_{NL}}$. If $\epsilon_H \ll 1$, $2(1 - n_s - 2\epsilon_H) \simeq 0.08$ for $n_s = 0.96$.

In addition, we are also interested in the trispectrum in this case. See the Appendix. A in detail.

3. Axion N-flation

In this section we focus on axion N-flation model [5] in which there are N_f inflaton fields and the potential of ϕ_i is given by

$$V_i = \Lambda_i^4 (1 - \cos \alpha_i), \quad (3.1)$$

where $\alpha_i = 2\pi\phi_i/f_i$ and f_i is the i-th axion decay constant. The ‘slow-roll’ parameters ϵ_i ’s are given by

$$\epsilon_i = \frac{2\pi^2}{f_i^2} \frac{1 + \cos \alpha_i}{1 - \cos \alpha_i}, \quad (3.2)$$

and

$$N_{,ii} = 1 - \frac{V_i V''_i}{V'^2_i} = \frac{1}{1 + \cos \alpha_i}. \quad (3.3)$$

Now the number of e-folds before the end of inflation becomes

$$N \simeq \sum_{i=1}^{N_f} \left(\frac{f_i}{2\pi} \right)^2 \ln \frac{2}{1 + \cos \alpha_i}. \quad (3.4)$$

Quoting the results in [5], the observables of axion N-flation are

$$P_\zeta = \frac{H^2}{8\pi^2} \sum_i \frac{1}{\epsilon_i}, \quad (3.5)$$

$$n_s = 1 - 2\epsilon_H - \frac{8\pi^2}{3H^2} \sum_i \frac{\Lambda_i^4}{f_i^2} \frac{1}{\epsilon_i} \bigg/ \sum_j \frac{1}{\epsilon_j}, \quad (3.6)$$

$$r_T = 16 \bigg/ \sum_i \frac{1}{\epsilon_i}, \quad (3.7)$$

$$f_{NL} = \frac{5}{6} \frac{r_T^2}{128} \sum_i \frac{1}{\epsilon_i} \frac{1}{1 + \cos \alpha_i}. \quad (3.8)$$

Using the results in the previous section, we can calculate the spectral index of f_{NL} for the axion N-flation, namely

$$n_{multi,i} = (1 - n_s - 2\epsilon_H) - \frac{8\pi^2}{3H^2} \frac{\Lambda_i^4}{f_i^2}, \quad (3.9)$$

$$n_{f,i} = 4\left(\frac{V_i}{V}\right)^2 \epsilon_i - 2\frac{V_i}{V} \epsilon_i, \quad (3.10)$$

and then

$$\begin{aligned} n_{f_{NL}} &= 2(1 - n_s - 2\epsilon_H) - \frac{16\pi^2}{3H^2} \sum_i \frac{\Lambda_i^4}{f_i^2} \frac{1}{\epsilon_i} \frac{1}{1 + \cos \alpha_i} \bigg/ \sum_j \frac{1}{\epsilon_j} \frac{1}{1 + \cos \alpha_j} \\ &+ 2 \sum_i \left(2\left(\frac{V_i}{V}\right)^2 - \frac{V_i}{V} \right) \frac{1}{1 + \cos \alpha_i} \bigg/ \sum_j \frac{1}{\epsilon_j} \frac{1}{1 + \cos \alpha_j}. \end{aligned} \quad (3.11)$$

The first and second lines in Eq.(3.11) are contributed by $n_{multi,i}$ and $n_{f,i}$ respectively. Since the second term in $n_{multi,i}$ is large compared to $n_{f,i}$ for $\alpha_i \simeq \pi$ which corresponds to a large f_{NL} , the spectral index of f_{NL} is mainly determined by $n_{multi,i}$.

3.1 Axion N-flation with the same potential for different inflatons

In this subsection we discuss the model with $\Lambda_i = \Lambda$ and $f_i = f$ for $\forall i$ in detail. Now the tensor-scalar ratio and f_{NL} become

$$r_T = \frac{32\pi^2}{f^2} \bigg/ \sum_i \frac{1 - \cos \alpha_i}{1 + \cos \alpha_i}, \quad (3.12)$$

$$f_{NL} = \frac{10\pi^2}{3f^2} \sum_i \frac{1 - \cos \alpha_i}{(1 + \cos \alpha_i)^2} \bigg/ \left(\sum_j \frac{1 - \cos \alpha_j}{1 + \cos \alpha_j} \right)^2, \quad (3.13)$$

and the spectral indices of scalar power spectrum and f_{NL} are simplified to be

$$n_s = 1 - 2\epsilon_H - \frac{8\pi^2\Lambda^4}{3H^2f^2}, \quad (3.14)$$

$$n_{f_{NL}} = \frac{f^2}{8\pi^2}(1 - n_s - 2\epsilon_H)^2 \sum_i \frac{1}{\epsilon_i}(1 - \cos \alpha_i) \Big/ \sum_j \frac{1}{\epsilon_j} \frac{1}{1 + \cos \alpha_j} - \frac{5r_T}{96} \frac{1}{f_{NL}}(1 - n_s - 2\epsilon_H), \quad (3.15)$$

where

$$\frac{\Lambda^4}{3H^2} = 1 \Big/ \sum_i (1 - \cos \alpha_i), \quad (3.16)$$

and

$$\epsilon_H = \frac{2\pi^2}{f^2} \sum_i \frac{1 - \cos 2\alpha_i}{2} \Big/ \left(\sum_j (1 - \cos \alpha_j) \right)^2. \quad (3.17)$$

We see that $n_{multi,i} = 0$ and $n_{f_{NL}}$ is expected to be small in this simple setup. We will show the smallness of $n_{f_{NL}}$ and extend the discussions in [5] in the following two subsubsections.

3.1.1 $N_* \simeq N_{tot}$

Assuming the initial conditions for α_i satisfy a uniform distribution, the number of e-folds before the end of inflation is roughly given by

$$N_{tot} \simeq \frac{\ln 2}{2\pi^2} f^2 N_f. \quad (3.18)$$

A large enough total number of e-folds, for example not less than 60, can be achieved as long as the number of inflatons is large enough, namely $N_f \gtrsim 1.7 \times 10^3 / f^2$.

Similar to [5], we consider the case with $N_* \simeq N_{tot}$ in this subsubsection, where N_* denotes the number of e-folds corresponding to the CMB scale. So we have

$$N_f f^2 \simeq \frac{2\pi^2 N_*}{\ln 2}, \quad (3.19)$$

and $\Lambda^4/(3H^2) \simeq 1/N_f$, $\epsilon_H \simeq \pi^2/(N_f f^2)$. The spectral index of the scale power spectrum is related to N_* by

$$n_s \simeq 1 - \frac{5 \ln 2}{N_*}, \quad (3.20)$$

which is the same as that in [5]. For $N_* \simeq 60$, $n_s \simeq 0.942$. The numerical results for $N_* \simeq N_{tot} \simeq 60$ and $f = 1/2$ are illustrated in Fig. 1. The spectral index of scalar power spectrum is nicely compatible with the estimation, and the tensor-scalar ratio is roughly $\mathcal{O}(10^{-5} \sim 10^{-3})$. As long as N_f is large enough, from Eqs. (3.12) and (3.13), both r_T and

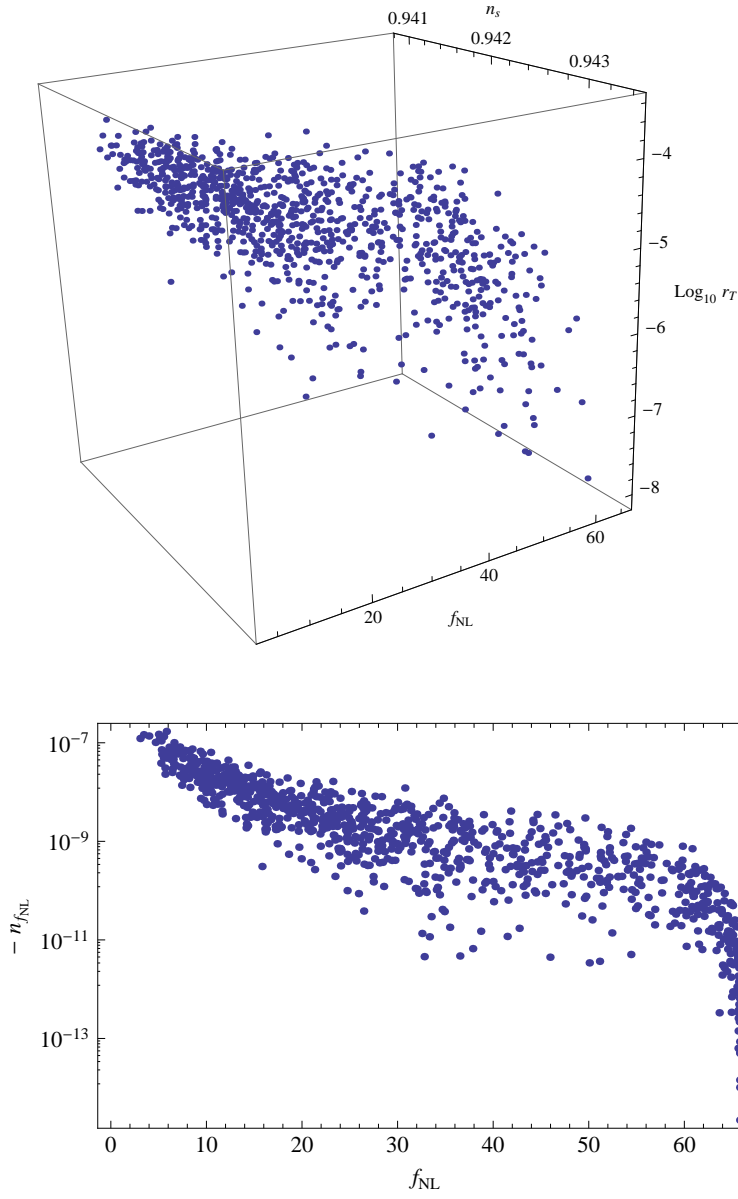


Figure 1: The observables of axion N-flation with the same potential for different inflatons. Here $f = 1/2$ and the initial conditions of α_i is assumed to be the uniform distribution.

f_{NL} are proportional to $1/f^2$. From the numerical results, we find that f_{NL} is bounded from above, namely

$$f_{NL} \lesssim \frac{5\pi^2}{3f^2}. \quad (3.21)$$

For $f \lesssim 1$, the value of f_{NL} can be of order $\mathcal{O}(10)$ which is large enough to be detected in the near future, but its tilt $n_{f_{NL}}$ ($-\mathcal{O}(10^{-7}) \sim -\mathcal{O}(10^{-14})$) is too small to be detected in this case.

3.1.2 A few inflatons stays around the hilltop

In [5], the authors proposed an alternative way to achieve a large f_{NL} . Roughly speaking, the summation of the function which is proportional to $1/\epsilon_i$ is dominated by those fields with the smallest ϵ_i , or equivalently staying around the hilltop. Suppose some number \bar{N} of fields have roughly comparable ϵ_i , of order $\bar{\epsilon}$ which is related to the angle $\bar{\alpha}$ by

$$\bar{\epsilon} \simeq \frac{\pi^2}{f^2}(1 + \cos \bar{\alpha}) \simeq \frac{\pi^2}{2f^2}(\pi - \bar{\alpha})^2, \quad (3.22)$$

with $(\pi - \bar{\alpha}) \ll 1$. In this case, P_ζ , r_T , and f_{NL} becomes

$$P_\zeta \simeq \frac{H^2}{8\pi^2} \frac{\bar{N}}{\bar{\epsilon}} \simeq \frac{H^2 \bar{N} f^2}{4\pi^4(\pi - \bar{\alpha})^2}, \quad (3.23)$$

$$r_T \simeq \frac{16\bar{\epsilon}}{\bar{N}} \simeq \frac{8\pi^2}{\bar{N}f^2}(\pi - \bar{\alpha})^2, \quad (3.24)$$

$$f_{NL} \simeq \frac{5\pi^2}{3\bar{N}f^2}. \quad (3.25)$$

Considering $\bar{N} \geq 1$,

$$f_{NL} \lesssim \frac{5\pi^2}{3f^2}, \quad (3.26)$$

which is the same as that in Sec. 3.1.1. Now the spectral index of f_{NL} is given by

$$n_{f_{NL}} \simeq -\frac{1 + \cos \bar{\alpha}}{2}(1 - n_s - 2\epsilon_H), \quad (3.27)$$

which is much smaller than $(1 - n_s - 2\epsilon_H)$. So the spectral index of f_{NL} is undetectable in this case as well.

Because the potential become very flat around the hilltop, one may worry that the classical motions of those inflatons staying around the hilltop are not reliable. Requiring that the classical motion of inflaton $\bar{\phi} = f\bar{\alpha}/2\pi$ per Hubble time be not less than its quantum fluctuation $H/2\pi$ yields

$$(\pi - \bar{\alpha}) \gtrsim \frac{3fH^3}{4\pi^2\Lambda^4} = 2\xi^{-1}\frac{H}{f}, \quad (3.28)$$

where $\xi = 8\pi^2\Lambda^4/(3H^2f^2) = (1 - n_s - 2\epsilon_H) \ll 1$.¹ On the other hand, from (3.23), $(\pi - \bar{\alpha})$ can be estimated by

$$(\pi - \bar{\alpha}) = \frac{\sqrt{\bar{N}}Hf}{2\pi^2\Delta_{\mathcal{R}}}, \quad (3.29)$$

where $\Delta_{\mathcal{R}} = \sqrt{P_\zeta}$. Therefore

$$\sqrt{\bar{N}}f^2 \gtrsim \frac{4\pi^2\Delta_{\mathcal{R}}}{\xi}. \quad (3.30)$$

¹In [5], the authors consider $|\phi_{hilltop} - \bar{\phi}| \gtrsim H/2\pi$, namely $(\pi - \bar{\alpha}) \gtrsim H/f$, which is much looser than what we require, where $\phi_{hilltop} \equiv f/2$.

Taking Eq.(3.25) into account, f_{NL} is bounded from above by

$$f_{NL} \lesssim \frac{5}{12} \frac{\xi}{\Delta_{\mathcal{R}}} / \sqrt{N}. \quad (3.31)$$

WMAP normalization [27] implies $P_{\zeta} = \Delta_{\mathcal{R}}^2 = 2.461 \times 10^{-9}$, and then $f_{NL} \lesssim 8 \times 10^3 \xi / \sqrt{N}$. Since $\xi \lesssim \mathcal{O}(10^{-2})$, $f_{NL} \lesssim 80 / \sqrt{N}$. The value of f_{NL} cannot be very large in this case.

3.2 Axion N-flation with different potential for different inflatons

In this subsection, we consider a class of axion N-flation in which $\Lambda_i = \Lambda$ for $\forall i$, but the decay constants can be different for different inflatons. For simplicity, we only focus on the cases with only two different decay constants.

3.2.1 One inflaton with different decay constant stays around the hilltop

In this subsubsection we consider a model with potentials

$$V(\phi_i) = \Lambda^4 (1 - \cos \alpha_i), \quad (3.32)$$

$$V(\bar{\phi}) = \Lambda^4 (1 - \cos \bar{\alpha}), \quad (3.33)$$

where $\alpha_i = 2\pi\phi_i/f$ for $i = 1, 2, \dots, N_f$ and $\bar{\alpha} = 2\pi\bar{\phi}/\bar{f}$. Here f is assumed to be different from \bar{f} . We consider $N_f \gg 1$ and the initial values of α_i satisfy a uniform distribution.² The total number of e-folds before the end of inflation is given by

$$N_{tot} \simeq \frac{\ln 2}{2\pi^2} f^2 N_f + \frac{\bar{f}^2}{4\pi^2} \ln \frac{2}{1 + \cos \bar{\alpha}}. \quad (3.34)$$

We assume $\bar{\phi}$ stays around the hilltop of its potential. Requiring that the classical motion of inflaton $\bar{\phi}$ is reliable leads to

$$(\pi - \bar{\alpha}_{ini}) \gtrsim \frac{3\bar{f}H^3}{4\pi^2\Lambda^4}, \quad (3.35)$$

where $\bar{\alpha}_{ini}$ denotes the initial value of $\bar{\alpha}$.

In this model, the Hubble parameter is roughly related to Λ by $H^2 \simeq N_f \Lambda^4 / 3$. Because the inflaton staying around the hilltop almost does not move, we also have

$$\epsilon_H \simeq \frac{\ln 2}{2} \left/ \left(N_{tot} - \frac{\bar{f}^2}{2\pi^2} \ln \frac{2}{\pi - \bar{\alpha}} \right) \right. . \quad (3.36)$$

The spectral index of power spectrum and f_{NL} become

$$n_s \simeq 1 - \frac{\ln 2}{N_{tot} - \frac{\bar{f}^2}{2\pi^2} \ln \frac{2}{\pi - \bar{\alpha}}} \left[1 + 4 \frac{c_r(N_f)N_f + \frac{4}{(\pi - \bar{\alpha})^2}}{c_r(N_f)N_f + (\bar{f}/f)^2 \frac{4}{(\pi - \bar{\alpha})^2}} \right], \quad (3.37)$$

$$n_{f_{NL}} \simeq \frac{8 \ln 2}{N_{tot} - \frac{\bar{f}^2}{2\pi^2} \ln \frac{2}{\pi - \bar{\alpha}}} \left[\frac{c_r(N_f)N_f + \frac{4}{(\pi - \bar{\alpha})^2}}{c_r(N_f)N_f + (\bar{f}/f)^2 \frac{4}{(\pi - \bar{\alpha})^2}} - \frac{d_r(N_f)N_f + \frac{8}{(\pi - \bar{\alpha})^4}}{d_r(N_f)N_f + (\bar{f}/f)^2 \frac{8}{(\pi - \bar{\alpha})^4}} \right], \quad (3.38)$$

²Thank C. T. Byrnes for the helpful discussion.

where

$$c_r(N) = \frac{1}{N} \sum_{i=1}^N \frac{1 - \cos \alpha_i}{1 + \cos \alpha_i}, \text{ and } d_r(N) = \frac{1}{N} \sum_{i=1}^N \frac{1 - \cos \alpha_i}{(1 + \cos \alpha_i)^2}, \quad (3.39)$$

which depend on the total number of points and the random choice of the series ‘ r ’ for $\alpha_i \in [0, \pi]$.

Similar to Sec. 3.1.1, we consider that the total number of e-folds corresponds to the CMB scale, namely $N_* \simeq N_{tot} \simeq 60$. Considering $\bar{f} \lesssim 1$, the total number of e-fold is mainly contributed by ϕ_i for $i = 1, 2, \dots, N_f$. If $f = \bar{f}$, $n_s \simeq 1 - 5 \ln 2 / N_*$ and $n_{f_{NL}} \simeq 0$ which are consistent with the results in Sec. 3.1.1.

For $\bar{f} < f$, the second term in the bracket of Eq. (3.37) is larger than 4 and then $n_s \lesssim 1 - 5 \ln 2 / N_*$. For $\bar{f} > f$, we find $n_s \gtrsim 1 - 5 \ln 2 / N_*$. In general, it is difficult to get the analytical estimation for the model with $f \neq \bar{f}$, but we can expect that the results should be quite different the model with the same decay constants. The numerical results are showed in Fig. 2. Here we keep $f = 1/2$ and $(\pi - \bar{\alpha}_{ini})/\pi = 2 \times 10^{-4}$ fixed and explore the results for two choices of \bar{f} . From Fig. 2, we see that a detectable negative $n_{f_{NL}}$ is possibly generated for $\bar{f} = 1/4$. We also check the requirement in (3.35) which is satisfied as long as $(\pi - \bar{\alpha})/\pi \gtrsim 2 \times 10^{-5}$ for $\bar{f} = 1/4$. Unfortunately, the spectral index $n_s (\lesssim 0.91)$ in the case with detectable $n_{f_{NL}}$ is small compared to WMAP 7yr data [27] ($n_s = 0.968 \pm 0.012$). See the green dots in Fig. 2. For $\bar{f} = 1$, the spectral index can fit the WMAP 7yr data nicely, but the scale dependence of f_{NL} becomes negligibly small. See the red dots in Fig. 2. This setup indicates that a large $n_{f_{NL}}$ can be achieved in axion N-flation.

3.2.2 A general axion N-flation with two kinds of inflatons

In this subsection we consider a more general axion N-flation model in which there are two kinds of inflatons who have different decay constants,

$$V_1(\phi_{1,i}) = \Lambda^4(1 - \cos \alpha_{1,i}), \quad (3.40)$$

$$V_2(\phi_{2,j}) = \Lambda^4(1 - \cos \alpha_{2,j}), \quad (3.41)$$

where $\alpha_{1,i} = 2\pi\phi_{1,i}/f_1$ for $i = 1, 2, \dots, N_1$, and $\alpha_{2,j} = 2\pi\phi_{2,j}/f_2$ for $j = 1, 2, \dots, N_2$. Here f_1 is assumed to be different from f_2 . We also consider $N_1, N_2 \gg 1$ and the initial values of $\alpha_{1,i}$ and $\alpha_{2,j}$ satisfy a uniform distribution respectively. In this model the total number of e-folds before the end of inflation is given by

$$N_{tot} \simeq \frac{\ln 2}{2\pi^2} (f_1^2 N_1 + f_2^2 N_2). \quad (3.42)$$

In this model, the Hubble parameter is related to Λ by $H^2 = (N_1 + N_2)\Lambda^4/3$ and the slow-roll parameter is given by

$$\epsilon_H = \frac{\pi^2}{(N_1 + N_2)^2} (N_1/f_1^2 + N_2/f_2^2). \quad (3.43)$$

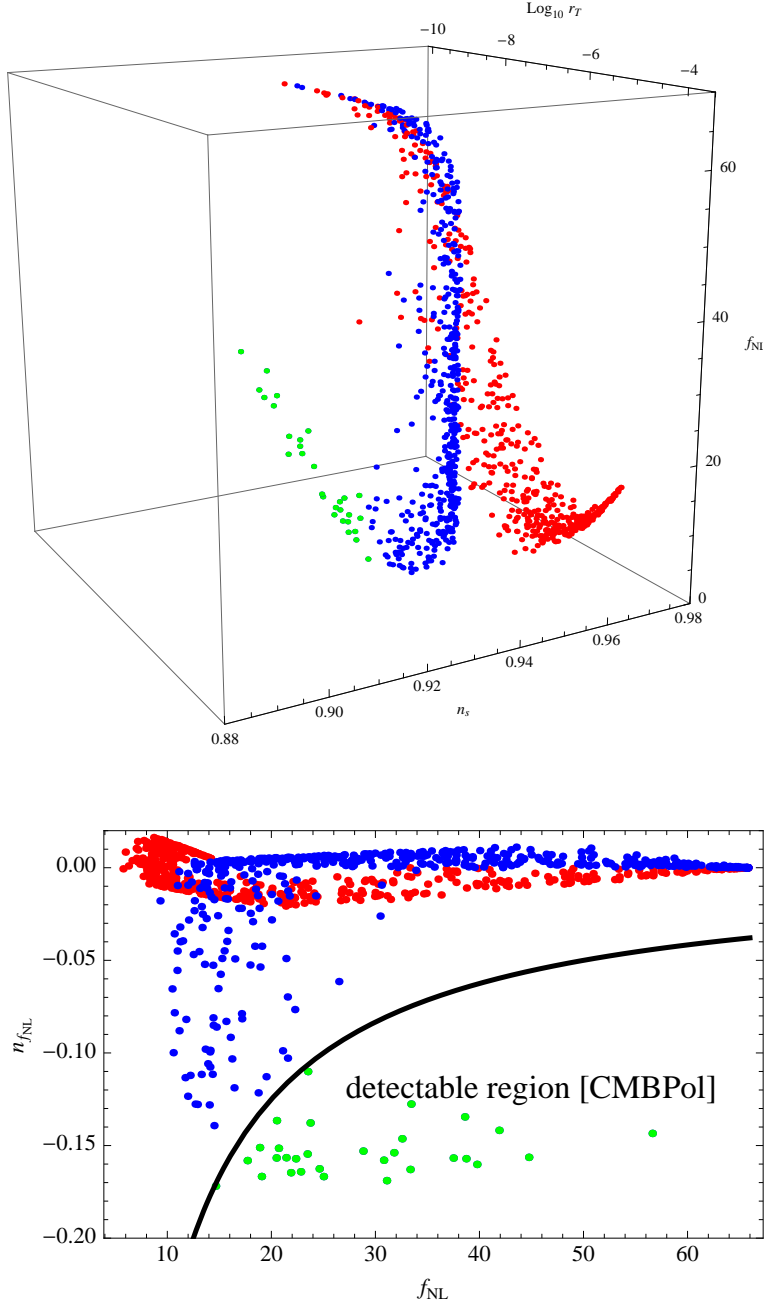


Figure 2: The observables of axion N-flation with different decay constants. Here we adopt $f = 1/2$ and $(\pi - \bar{\alpha}_{ini})/\pi = 2 \times 10^{-4}$. The blue and green dots correspond to $\bar{f} = 1/4$ and the red dots corresponds to $\bar{f} = 1$. The green dots illustrate the case with detectable scale dependence of f_{NL} .

Similarly, we can also derive the spectral index of power spectrum and f_{NL} , namely

$$n_s \simeq 1 - \frac{2\pi^2}{(N_1 + N_2)^2} \left(\frac{N_1}{f_1^2} + \frac{N_2}{f_2^2} \right) - \frac{8\pi^2}{N_1 + N_2} \frac{c_{r_1}(N_1)N_1 + c_{r_2}(N_2)N_2}{c_{r_1}(N_1)f_1^2N_1 + c_{r_2}(N_2)f_2^2N_2}, \quad (3.44)$$

$$n_{f_{NL}} \simeq \frac{16\pi^2}{N_1 + N_2} \left[\frac{c_{r_1}(N_1)N_1 + c_{r_2}(N_2)N_2}{c_{r_1}(N_1)f_1^2N_1 + c_{r_2}(N_2)f_2^2N_2} - \frac{d_{r_1}(N_1)N_1 + d_{r_2}(N_2)N_2}{d_{r_1}(N_1)f_1^2N_1 + d_{r_2}(N_2)f_2^2N_2} \right]. \quad (3.45)$$

Again, if $f_1 = f_2$, the scale dependence of f_{NL} is negligibly small and our results recover those in Sec. 3.1.1.

For the case with $f_1 \neq f_2$, we need to adopt numerical method to explore it. Considering $N_* \simeq N_{tot} \simeq 60$, our numerical results are illustrated in Fig. 3. Here we adopt

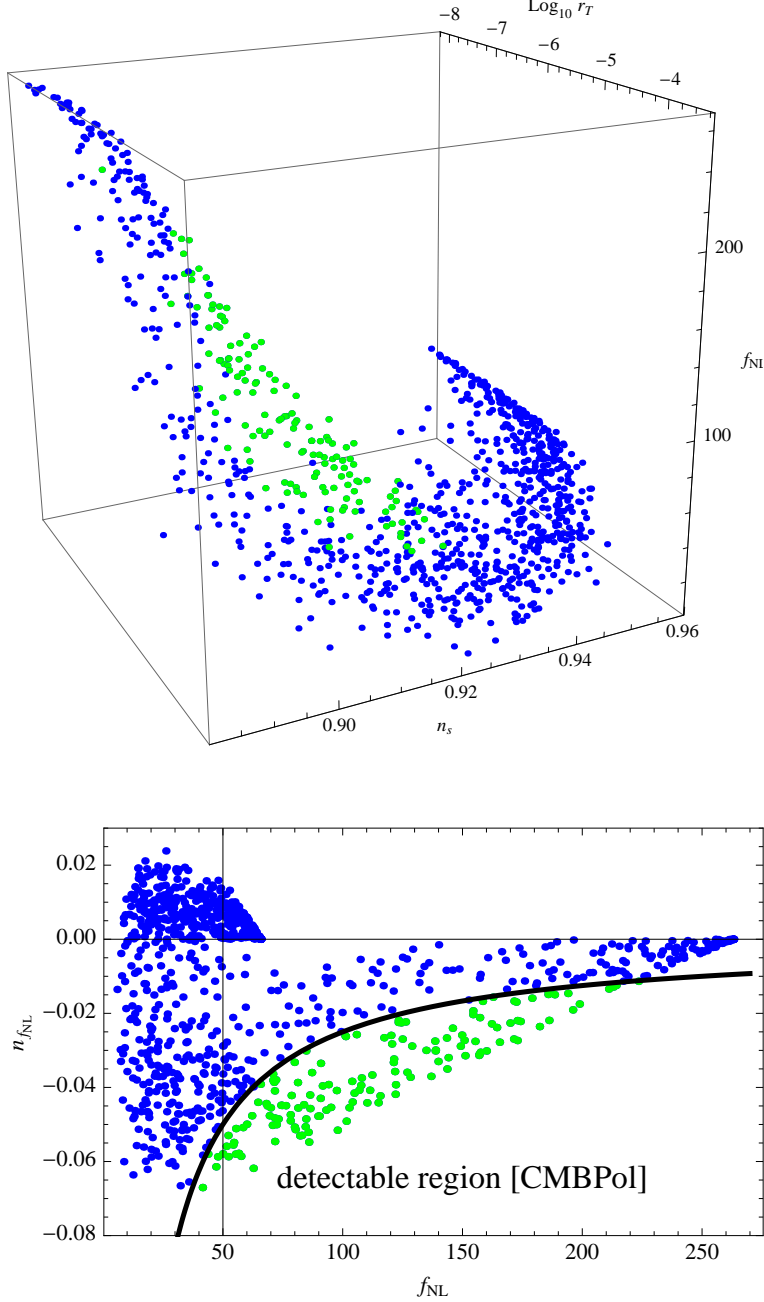


Figure 3: The observables of a general axion N-flation with different decay constants. Here we adopt $f_1 = 1/2$ and $f_2 = 1/4$. The green dots illustrate the case with detectable scale dependence of f_{NL} .

$f_1 = 1/2$, $N_1 = 4800$, $f_2 = 1/4$ and $N_2 = 8000$. We see that a detectable scale dependence of f_{NL} can be obtained.

4. Discussions

In this paper we work out the spectral index of f_{NL} in general N-flation model by adopting the horizon-crossing approximation. For an instance, we focus on the axion N-flation proposed in [5] and we find that f_{NL} can be large enough to be detected in the near future.

- All inflatons have the same potential. In the case with the total number of e-folds corresponding to the CMB scale, f_{NL} is roughly of order $\mathcal{O}(10)$ for a uniform distribution of initial angles if the decay constant f is around the Planck scale. Our numerical results indicate that f_{NL} is bounded from above by $5\pi^2/3f^2$, and $\tau_{NL} \simeq (2\pi/f)^{2/3}f_{NL}^{5/3}$. Another way to obtain a large f_{NL} is that a few inflatons stay around the hilltop and the summations in the formula of f_{NL} is mainly contributed by these inflatons. Typically f_{NL} is less than one hundred in the second case. In both cases the tilt of f_{NL} is negligibly small. The smallness of $n_{f_{NL}}$ in these two case comes from the accident cancellation in $n_{multi,i}$ because all inflatons have the same potential.

- Different inflatons have different decay constants. In this kind of model, the accident cancellation in $n_{multi,i}$ does not happen and a negative detectable $n_{f_{NL}}$ can be obtained. It implies that a scale independent f_{NL} is not generic prediction of N-flation. A more general axion N-flation is left to be studied in the future.

In addition, we also study the trispectrum for N-flation in the Appendix. A. We find that there is a universal relation between g_{NL} and τ_{NL} , namely $g_{NL} \simeq 25\tau_{NL}/27$, for the cases with large f_{NL} . But the relation between f_{NL} and τ_{NL} depends on detail of the model.

If there is only one inflaton field, the axion N-flation becomes natural inflation. The slow-roll parameters are roughly given by M_p^2/f^2 and then the slow-roll conditions are satisfied if $f > M_p$. However such a large decay constant cannot be achieved in some fundamental theories, such as string theory [28, 29]. In [30] the N-flation is supposed to realize a slow-roll inflation even when $f < M_p$ by calling multi inflaton fields. On the other hand, in [31] we proposed a conjecture that the effective gravity scale should be $\Lambda_G = M_p/\sqrt{N_f}$ for the case with multi scalar fields, not M_p . If the decay constant of axion field is required to be smaller than Λ_G , namely $N_f f^2 < 1$, the total number of e-folds is less than one and the axion N-flation fails.

Acknowledgments

We would like to thank S. A. Kim and A. R. Liddle for useful discussions. QGH is supported by the project of Knowledge Innovation Program of Chinese Academy of Science and a grant from NSFC.

A. The trispectrum in N-flation

From Eq. (1.1), the size of trispectrum is characterized by two parameters τ_{NL} and g_{NL} which are defined by

$$\tau_{NL} = \sum_{i,j,k} N_{,ij} N_{,ik} N_{,j} N_{,k} \Big/ (\sum_l N_{,l}^2)^3, \quad (\text{A.1})$$

$$g_{NL} = \frac{25}{54} \sum_{i,j,k} N_{,ijk} N_{,i} N_{,j} N_{,k} \Big/ (\sum_l N_{,l}^2)^3. \quad (\text{A.2})$$

For N-flation, the above two equations are simplified to be

$$\tau_{NL} = \sum_i N_{,ii}^2 N_{,i}^2 \Big/ (\sum_l N_{,l}^2)^3, \quad (\text{A.3})$$

$$g_{NL} = \frac{25}{54} \sum_i N_{,iii} N_{,i}^3 \Big/ (\sum_l N_{,l}^2)^3, \quad (\text{A.4})$$

where

$$N_{,iii} = 2 \frac{V_i V_i''^2}{V_i'^3} - \frac{V_i''}{V_i'} - \frac{V_i V_i'''}{V_i'^2}. \quad (\text{A.5})$$

We remind the readers that the value of τ_{NL} is bounded from below by $(\frac{6}{5}f_{NL})^2$. For the axion N-flation, we have

$$N_{,iii} = \frac{2\pi}{f_i} \frac{\sin \alpha_i}{(1 + \cos \alpha_i)^2}. \quad (\text{A.6})$$

Therefore

$$\tau_{NL} = \sum_i \frac{f_i^2}{4\pi^2} \frac{1 - \cos \alpha_i}{(1 + \cos \alpha_i)^3} \Big/ \left(\sum_j \frac{f_j^2}{4\pi^2} \frac{1 - \cos \alpha_j}{1 + \cos \alpha_j} \right)^3, \quad (\text{A.7})$$

$$g_{NL} = \frac{25}{54} \sum_i \frac{f_i^2}{4\pi^2} \frac{(1 - \cos \alpha_i)^2}{(1 + \cos \alpha_i)^3} \Big/ \left(\sum_j \frac{f_j^2}{4\pi^2} \frac{1 - \cos \alpha_j}{1 + \cos \alpha_j} \right)^3. \quad (\text{A.8})$$

For the case with $f_i = f$ for $\forall i$, both τ_{NL} and g_{NL} are proportional to $1/f^4$. Similar to Sec. 3.1.1, we consider $N_* \simeq N_{tot} \simeq 60$ and the numerical results for $f = 1/2$ are showed in Fig. 4. Fitting the numerical results, we find that τ_{NL} and g_{NL} are roughly related to f_{NL} and f by

$$\tau_{NL} \simeq \left(\frac{2\pi}{f} \right)^{2/3} f_{NL}^{5/3}, \quad g_{NL} \simeq \frac{25}{27} \tau_{NL}. \quad (\text{A.9})$$

Roughly speaking, $f_{NL} \sim 1/f^2$ and then $g_{NL} \simeq \frac{25}{27} \tau_{NL} \sim 1/f^4$ which is consistent with our estimation. On the other hand, combining (A.9) and $\tau_{NL} \geq (\frac{6}{5}f_{NL})^2$, we get $f_{NL} \lesssim 4\pi^2/3f^2$ which is roughly the same as our results in Sec. 3.1.1.

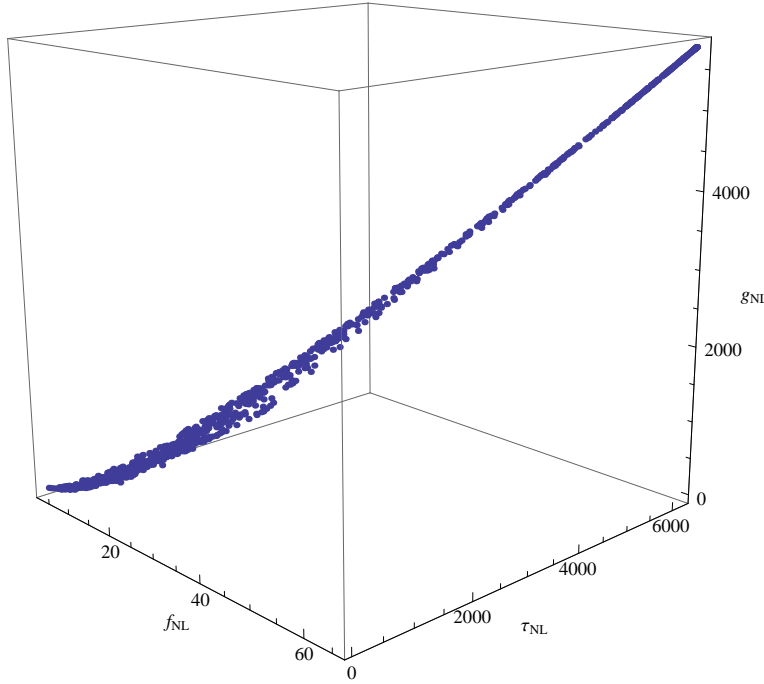


Figure 4: The non-Gaussianity parameters of axion N-flation for the uniform distribution of α_i when $f = 1/2$. Here we consider the case with $N_* \simeq N_{tot} \simeq 60$.

The non-Gaussianity parameters of trispectrum for the case in which a few inflatons stay around the hilltop were worked out in [5]: $\tau_{NL} = (\frac{6}{5}f_{NL})^2$ and $g_{NL} = \frac{25}{27}\tau_{NL}$, which are different from the previous case unless $f_{NL} \simeq 4\pi^2/3f^2$.

The non-Gaussianity parameters for the model in Sec. 3.2.1 are showed in Fig. 5. Fitting the green dots corresponding to a detectable scale dependence of f_{NL} , we find

$$\tau_{NL} \simeq 7.78f_{NL}^{1.77}, \quad g_{NL} \simeq \frac{25}{27}\tau_{NL}. \quad (\text{A.10})$$

Fitting the blue and red dots respectively, we reach the same results

$$\tau_{NL} \simeq 8.28f_{NL}^{1.59}, \quad g_{NL} \simeq \frac{25}{27}\tau_{NL}. \quad (\text{A.11})$$

Here the relations between τ_{NL} and f_{NL} depend on the choice of the parameters, such as f , \bar{f} and $\bar{\alpha}$, in the model.

Similarly we also investigate the non-Gaussianity parameters for the model in Sec. 3.2.2. See the numerical results in Fig. 6. Fitting the blue and green dots respectively, we find

$$\tau_{NL} \simeq 7.16f_{NL}^{1.72}, \quad g_{NL} \simeq \frac{25}{27}\tau_{NL}. \quad (\text{A.12})$$

To summarize, there is a universal relation between g_{NL} and τ_{NL} , namely $g_{NL} \simeq \frac{25}{27}\tau_{NL}$. The value of g_{NL} is the same order of τ_{NL} . But the relations between τ_{NL} and f_{NL} are different for different cases.

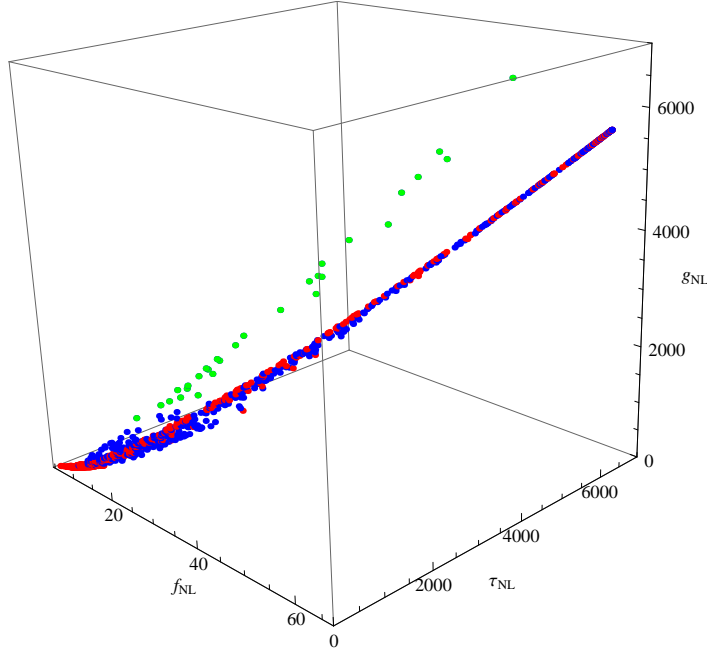


Figure 5: The non-Gaussianity parameters in the axion N-flation where N_f inflatons have the same decay constant f , and the decay constant of another inflaton is \bar{f} . Here we consider the case with $N_* \simeq N_{tot} \simeq 60$ and adopt $f = 1/2$ and $(\pi - \bar{\alpha}_{ini})/\pi = 2 \times 10^{-4}$. The blue and green dots correspond to $\bar{f} = 1/4$ and the red dots corresponds to $\bar{f} = 1$. The green dots illustrate the case with detectable scale dependence of f_{NL} .

References

- [1] J. M. Maldacena, “Non-Gaussian features of primordial fluctuations in single field inflationary models,” *JHEP* **0305**, 013 (2003) [arXiv:astro-ph/0210603].
- [2] A. A. Starobinsky, “Multicomponent de Sitter (Inflationary) Stages and the Generation of Perturbations,” *JETP Lett.* **42** (1985) 152;
M. Sasaki and E. D. Stewart, “A General Analytic Formula For The Spectral Index Of The Density Perturbations Produced During Inflation,” *Prog. Theor. Phys.* **95**, 71 (1996) [arXiv:astro-ph/9507001];
M. Sasaki and T. Tanaka, “Super-horizon scale dynamics of multi-scalar inflation,” *Prog. Theor. Phys.* **99**, 763 (1998) [arXiv:gr-qc/9801017];
D. H. Lyth, K. A. Malik and M. Sasaki, “A general proof of the conservation of the curvature perturbation,” *JCAP* **0505**, 004 (2005) [arXiv:astro-ph/0411220];
D. H. Lyth and Y. Rodriguez, “The inflationary prediction for primordial non-gaussianity,” *Phys. Rev. Lett.* **95**, 121302 (2005) [arXiv:astro-ph/0504045].
- [3] C. T. Byrnes, K. Y. Choi and L. M. H. Hall, “Conditions for large non-Gaussianity in two-field slow-roll inflation,” *JCAP* **0810**, 008 (2008) [arXiv:0807.1101 [astro-ph]].
- [4] C. T. Byrnes, K. Y. Choi and L. M. H. Hall, “Large non-Gaussianity from two-component hybrid inflation,” *JCAP* **0902**, 017 (2009) [arXiv:0812.0807 [astro-ph]].
- [5] S. A. Kim, A. R. Liddle and D. Seery, “Non-gaussianity in axion Nflation models,” arXiv:1005.4410 [astro-ph.CO].

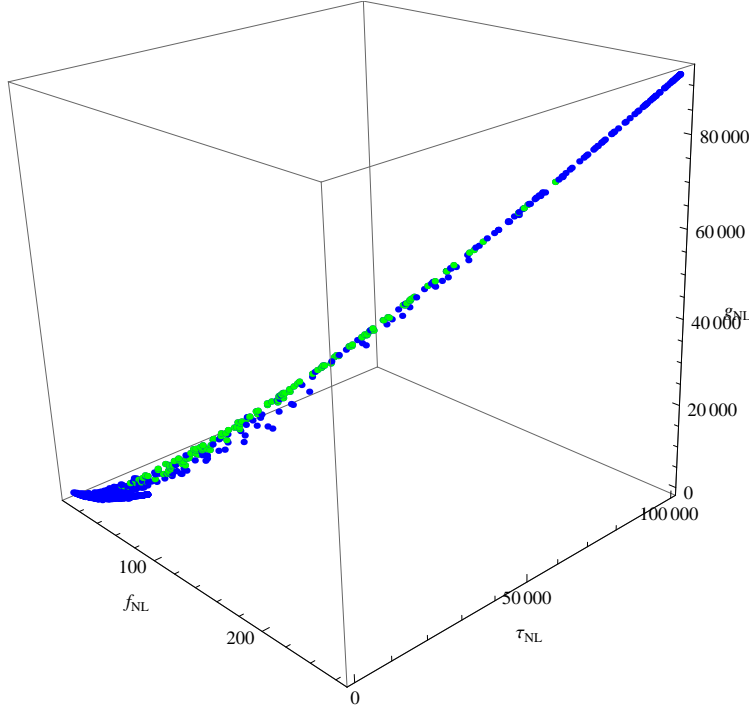


Figure 6: The non-Gaussianity parameters in the general axion N-flation with two kinds of inflatons. Here we consider the case with $N_* \simeq N_{tot} \simeq 60$. The green dots illustrate the case with detectable scale dependence of f_{NL} .

- [6] D. H. Lyth, “Generating the curvature perturbation at the end of inflation,” JCAP **0511**, 006 (2005) [arXiv:astro-ph/0510443].
- [7] M. Sasaki, “Multi-brid inflation and non-Gaussianity,” Prog. Theor. Phys. **120**, 159 (2008) [arXiv:0805.0974 [astro-ph]].
- [8] Q. G. Huang, “A geometric description of the non-Gaussianity generated at the end of multi-field inflation,” JCAP **0906**, 035 (2009) [arXiv:0904.2649 [hep-th]].
- [9] F. Vernizzi and D. Wands, “Non-Gaussianities in two-field inflation,” JCAP **0605**, 019 (2006) [arXiv:astro-ph/0603799].
- [10] S. A. Kim and A. R. Liddle, “Nflation: Multi-field inflationary dynamics and perturbations,” Phys. Rev. D **74**, 023513 (2006) [arXiv:astro-ph/0605604].
- [11] Y. S. Piao, “On perturbation spectra of N-flation,” Phys. Rev. D **74**, 047302 (2006) [arXiv:gr-qc/0606034];
R. G. Cai, B. Hu and Y. S. Piao, “Entropy Perturbations in N-flation Model,” Phys. Rev. D **80**, 123505 (2009) [arXiv:0909.5251 [astro-ph.CO]].
- [12] S. A. Kim and A. R. Liddle, “Nflation: Non-gaussianity in the horizon-crossing approximation,” Phys. Rev. D **74**, 063522 (2006) [arXiv:astro-ph/0608186].
- [13] T. Battefeld and R. Easther, “Non-gaussianities in multi-field inflation,” JCAP **0703**, 020 (2007) [arXiv:astro-ph/0610296].

- [14] D. Battefeld and T. Battefeld, “Non-Gaussianities in N-flaton,” JCAP **0705**, 012 (2007) [arXiv:hep-th/0703012].
- [15] D. Langlois, F. Vernizzi and D. Wands, “Non-linear isocurvature perturbations and non-Gaussianities,” JCAP **0812**, 004 (2008) [arXiv:0809.4646 [astro-ph]].
- [16] D. J. Mulryne, D. Seery and D. Wesley, “Moment transport equations for the primordial curvature perturbation,” arXiv:1008.3159 [astro-ph.CO].
- [17] T. Wang, “Note on Non-Gaussianities in Two-field Inflation,” arXiv:1008.3198 [astro-ph.CO].
- [18] T. Suyama, T. Takahashi, M. Yamaguchi and S. Yokoyama, “On Classification of Models of Large Local-Type Non-Gaussianity,” arXiv:1009.1979 [astro-ph.CO].
- [19] E. Sefusatti, M. Liguori, A. P. S. Yadav, M. G. Jackson and E. Pajer, “Constraining Running Non-Gaussianity,” JCAP **0912**, 022 (2009) [arXiv:0906.0232 [astro-ph.CO]].
- [20] C. T. Byrnes, S. Nurmi, G. Tasinato and D. Wands, “Scale dependence of local f_{NL} ,” JCAP **1002**, 034 (2010) [arXiv:0911.2780 [astro-ph.CO]].
- [21] C. T. Byrnes, M. Gerstenlauer, S. Nurmi, G. Tasinato and D. Wands, “Scale-dependent non-Gaussianity probes inflationary physics,” arXiv:1007.4277 [astro-ph.CO].
- [22] C. T. Byrnes, K. Enqvist and T. Takahashi, “Scale-dependence of Non-Gaussianity in the Curvaton Model,” arXiv:1007.5148 [astro-ph.CO].
- [23] Q. G. Huang, “Negative spectral index of f_{NL} in the axion-type curvaton model,” arXiv:1008.2641 [astro-ph.CO].
- [24] A. Riotto and M. S. Sloth, “Strongly Scale-dependent Non-Gaussianity,” arXiv:1009.3020 [astro-ph.CO].
- [25] D. H. Lyth and A. Riotto, “Particle physics models of inflation and the cosmological density perturbation,” Phys. Rept. **314**, 1 (1999) [arXiv:hep-ph/9807278].
- [26] M. Sasaki and E. D. Stewart, “A General Analytic Formula For The Spectral Index Of The Density Perturbations Produced During Inflation,” Prog. Theor. Phys. **95**, 71 (1996) [arXiv:astro-ph/9507001].
- [27] E. Komatsu *et al.*, “Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation,” arXiv:1001.4538 [astro-ph.CO].
- [28] N. Arkani-Hamed, H. C. Cheng, P. Creminelli and L. Randall, “Extranatural inflation,” Phys. Rev. Lett. **90**, 221302 (2003) [arXiv:hep-th/0301218].
- [29] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, “On the possibility of large axion decay constants,” JCAP **0306**, 001 (2003) [arXiv:hep-th/0303252].
- [30] S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, “N-flaton,” JCAP **0808**, 003 (2008) [arXiv:hep-th/0507205].
- [31] Q. G. Huang, “Weak Gravity Conjecture for the Effective Field Theories with N Species,” Phys. Rev. D **77**, 105029 (2008) [arXiv:0712.2859 [hep-th]].